International Journal of Theoretical Physics, Vol. 33, No. 1, 1994

Quantum Physics as a Particular Domain of Probabilistic Physics

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Received July 22, 1993

Quantum physics (QP) is meant as a whole science having both theoretical and experimental parts. The subjects of these parts in any science are entirely different. The experimental part deals with really existing particular objects (*concrete objects*), whereas the theoretical part refers to the so-called *abstract objects* which are used in our considerations only. The necessity of a strict distinction between concrete and abstract objects is a crucial *key methodological principle* (KMP). This principle allows one to construct the science of probability (*probabilistics*) whose theoretical and experimental parts are, respectively, *probability theory* and *experimental statistics.* Probabilistics suggests two methods of solving probabilistic problems: the *classical method* and the *quantum approach.* The application of probabilistics to physics leads to *probabilistic physics,* whose two interconnected particular domains, *classical statistical physics* (CSP) and QP, result, respectively, from the treatment of macrosystems by the classical method and of microsystems by the quantum approach. The mathematical peculiarities of QP stem from the pertinent ones in probabilistics itself. Having been constructed as a particular domain of probabilistic physics, QP needs no artificial interpretation. Many quantum-related issues and paradoxes are thereby easily settled.

1. INTRODUCTION

In what follows quantum physics (QP) is meant as a whole science having both theoretical and experimental parts. The theoretical part of QP needs no artificial interpretation, for its subject matter is determined by the science itself—it deals with probabilistic considerations. The experimental part of QP involves the accumulation of experimental statistical data, in particular, by measuring values of pertinent quantities. Thus, measurements belong to the experimental part of QP, not to its theoretical part.

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0020-7748/94/0100-0031$07.00/0

A point of extreme importance is that the theoretical and experimental parts of any science (QP included) refer to entirely different, though tightly interconnected, subjects. The subjects of the experimental part are really existing particular objects on which experiments are carried out. They have been called by me *concrete objects*. The subjects of the theoretical part are meant to be devoid of the individual features that distinguish concrete objects from each other but are unessential in theoretical considerations. I have named them *abstract objects* (Mayants, 1973, 1984). Thus, concrete objects are ones we actually deal with in practice, whereas abstract objects are those we use in our considerations only. The necessity of a strict distinction between concrete objects and abstract objects, in order to avoid any misunderstanding and confusion, is a crucial *key methodological principle* (KMP). This principle has made it possible to construct the science of probability (*probabilistics*), whose theoretical and experimental parts are, respectively, *probability theory* and *experimental statistics*. In probabilistics the notion of *probability* is defined explicitly, and the interconnection between its two parts reveals itself, in particular, in an approximate equality of probabilities and pertinent experimental statistical frequencies at large enough number of random tests.

A mere application of probabilistics to physics leads to a science which I have called *probabilistic physics*, whose two particular interconnected domains are *classical statistical physics* (CSP) and QP. A straightforward construction of QP (and CSP) can thus be represented as:

$$\begin{tikzcd} & & & \text{CSP} \arrow[dd, dash, dashed] \\ \text{KMP} \arrow[r] & \text{Probabilistics} \arrow[r] & \text{Probabilistic physics} \arrow[ur] \arrow[dr] & \\ & & & \text{QP} \end{tikzcd}$$

Such way of constructing QP (as a whole science) eliminates many misunderstandings, misconceptions, paradoxes, etc., known to abound in conventional quantum physics.

2. KMP

The concept of concrete objects is a primary one which cannot be reduced to any simpler concepts. It refers to fully determined, really existing objects—the subjects of experimentation, in particular.

Consider a certain set A of concrete objects a such that $a \in A$ if and only if it has the value $f_0 \equiv f$ of the property t_0. We assume that elements of A have a set $T = \{t_0, t_1, \ldots, t_n\}$ of properties, and each property t_i has a set θ_i of values f_i, one and only one value $f_i \in \theta_i$ corresponding to every concrete object $a \in A$ (t_0 has only one value f for all $a \in A$). Hence, different concrete objects differ in the values of at least one of their properties.

Any subset $t_{ij\ldots k} = (t_i, t_j, \ldots, t_k) \subseteq T$ $(0 \le i < j < \cdots < k \le n)$ is also a property (combined), which has a set $\theta_{ij\ldots k}$ of values $f_{ij\ldots k} = (f_i, f_j, \ldots, f_k)$ where $f_i \in \theta_i$, $f_j \in \theta_j, \ldots, f_k \in \theta_k$. Let us divide A into classes, in accordance with the values of every property $t_{ij\ldots k}$. The class $A(f_{ij\ldots k})$ corresponding to $f_{ij\ldots k}$ contains all those $a \in A$ which have the values $f_i, f_j, \ldots, f_k$ of the properties $t_i, t_j, \ldots, t_k$, respectively. In particular, the class $A(f) \equiv A$ contains all $a \in A$. The following equation is evident:

$$A(f_{ij\ldots k}) = \bigcup_{\theta_{lm\ldots p}} A(f_{ij\ldots klm\ldots p}) \tag{1}$$

where $\theta_{lm\ldots p}$ is a set of values $f_{lm\ldots p} = (f_l, f_m, \ldots, f_p)$ of the property $t_{lm\ldots p}$ with $f_l \in \theta_l$, $f_m \in \theta_m, \ldots, f_p \in \theta_p$.

$\theta_{ij\ldots k}$ and the set $A(t_{ij\ldots k})$ of the classes $A(f_{ij\ldots k})$ are in one-to-one correspondence. Let us map the set $\bigcup_{ij\ldots k} A(t_{ij\ldots k})$, $i \ge 0$, of all the classes onto the set $\theta = \bigcup_{ij\ldots k} \theta_{ij\ldots k}$, $i \ge 0$, of all the values of the properties of concrete objects of A. Then $f_{ij\ldots k}$ is the image of the class $A(f_{ij\ldots k}) \subseteq A$, and, in particular, f is the image of the whole set A. This fact allows us to introduce a strict definition of abstract objects.

Definition 1. Abstract objects are elements of the set θ.

Thus $f_{ij\ldots k}$, the image of the class $A(f_{ij\ldots k})$, is the abstract object corresponding to this class of concrete objects. Only definite values of some of the properties of concrete objects of A refer to every abstract object, the enumeration of which fully determines the abstract object both in essence and name.

Concrete objects and abstract objects, though being of different nature, may have similar names. This fact often causes confusion—a mixup of concrete and abstract objects—which, in turn, leads to misunderstandings, paradoxes, etc. Hence *the necessity to distinguish strictly between concrete objects and abstract objects*, which is the above KMP. Note also that questions which may be asked about concrete objects and abstract objects are entirely different. Questions about exact values of properties of a concrete object are legitimate in principle, for a concrete object does have them. But questions concerning exact values of properties of an abstract object, which do not appear in its designation (its name), are irrelevant in principle. The only questions one may ask should refer to the distributions of those values for the class whose image the abstract object is. The related problems belong to probabilistics.

3. PROBABILISTICS

Consider in the class $A(f_{ij\ldots k}) \subseteq A$ the subset

$$A'(f_{ij\ldots k}; \theta'_{lm\ldots p}) = \bigcup_{\theta'_{lm\ldots p}} A(f_{ij\ldots klm\ldots p})$$

We suppose that all sets $A'(f_{ij\ldots k}; \theta'_{lm\ldots p})$ are measurable (L). The problem of finding their measures can be solved in some particular cases in many ways. There is, however, one general experimental way which allows one to do so, first, with an accuracy to an unknown factor and, second, merely approximately. This procedure, connected with the notion of "accident," can be called *the statistical method.*

The essence of the method is that a set A of concrete objects is subjected to a large number of *random tests,* that is, tests by which some concrete object of A is revealed accidentally. Then a general experimental observation is used, which follows from a huge body of statistical data accumulated in experimental statistics and represents, hence, the *basic phenomenon of probabilistics.* This observation is taken as the primary principal premise of probabilistics, which reads:

Axiom 1. If a very large number of random tests is made, the number of tests revealing concrete objects of each subset of a set A is *approximately* proportional to the measure of this subset.

In order for this axiom to be applicable, the tests *must* really be of a random nature, that is, they must actually lead to accidental results.

Thus it is precisely the *randomness* of tests that is essential to the statistical method, for it ensures the possibility of an experimental determination of the ratios between the measures of subsets of the set A. But these ratios themselves are quite definite quantitative characteristics of the structure of the set A and bear no relation to randomness.

Using the statistical method, one cannot predict, of course, the results of each separate random test. But if the ratio of the measure (mA') of some subset $A' \subset A$ to the measure (mA) of the set A is near unity, one can suppose with great confidence that as a result of a single random test just one of the concrete objects $a \in A'$ will be revealed. The larger mA'/mA, the greater this confidence, which brings us right to the definition of *probability* as the degree of confidence in the correctness of the assumption that a single random test made on a set A of concrete objects will reveal some concrete object of the subset $A' \subset A$, which can be estimated by the ratio mA'/mA. Although probability in the indicated sense has a subjective implication, this ratio is an objective quantitative characteristic of the structure of the set A.

At the same time, with a sufficiently large number of random tests, the statistical frequency of revealing concrete objects of A' must also be approximately equal to mA'/mA. Hence, the above probability approximately coincides numerically with the pertinent frequency (often called *statistical probability*). This allows us to give an explicit (preliminary) definition of probability.

Recall, first, that mA' is the measure of the set of concrete objects, which corresponds to the abstract object f "having" the set $\theta'_{12...n}$ of values of the property $t_{12...n}$, and mA is the measure of the whole set A of concrete objects, which corresponds to the abstract object f itself. Further, any class $A(f_{ij...k})$ can be taken as a new initial set of concrete objects, since the property $t_{ij...k}$ has one definite value $f_{ij...k}$ only on the whole class. In view of this, *probability* as an *objective* quantitative characteristic of the structure of the set $A(f_{ij...k})$ can be defined as follows.

Definition 2. The *probability* that for the abstract object $f_{ij...k}$ a value $f_{lm...p}$ of the property $t_{lm...p}$ belongs to the set $\theta'_{lm...p} \subseteq \theta_{lm...p}$ is the ratio of the measure of $A'(f_{ij...k}; \theta'_{lm...p})$ to the measure of $A(f_{ij...k})$.

Using this definition, one need not distinguish absolute from conditional probabilities. According to Definition 2, probability concerns abstract objects and bears no relation to accident. However, raising probabilistic problems itself makes sense only when there exists the possibility of revealing some concrete object of a set A by carrying out a random test. Usually one talks about the probability of some *random event*. It can be shown, however, that Definition 2 conforms fully to ordinary usage (Mayants, 1973, 1984).

Definition 2 is only valid when A is given and measure (L) can be introduced in it. But in most cases to which probabilistics applies neither supposition is fulfilled. Only the set T of properties (together with their values) and, hence, also the set θ of abstract objects are always given. Thus, a general definition is needed covering any experimentally verifiable case. This has been done by introducing statistically adequate sets (Mayants, 1973, 1984). The point is that the initial set A can be replaced by a measurable (L) concrete set $A^{(a)}$ which is adequate to it (Mayants, 1973, 1984). Then the specificity of probabilistics allows one to formulate its second (and last) axiom.

Axiom 2. For any set A of concrete objects which can be subjected to random tests there exists at least one set $A^{(a)}$ which is adequate to it.

With this axiom in mind, the general definition of probability can finally be put as follows.

Definition 2a. The *probability* that for the abstract object $f_{ij\ldots k}$ a value $f_{lm\ldots p}$ of the property $t_{lm\ldots p}$ belongs to the set $\theta'_{lm\ldots p} \subseteq \theta_{lm\ldots p}$ is the ratio of the measure of $A^{(a)}(f^{(a)}_{ij\ldots k};\theta^{(a)\prime}_{lm\ldots p})$ to the measure of $A^{(a)}(f^{(a)}_{ij\ldots k})$, where $A^{(a)}$ is any set adequate to A.

Denoting this probability by $P(f_{ij\ldots k};\theta'_{lm\ldots p})$, Definition 2a can be presented as

$$P(f_{ij\ldots k};\theta'_{lm\ldots p}) = mA'(f_{ij\ldots k};\theta'_{lm\ldots p})/mA(f_{ij\ldots k}) \tag{2}$$

[For the sake of simplicity the upper indices a in (2) are omitted, but they are implied.] If A is given and measurable (L), Axiom 2 is satisfied, Definitions 2a and 2 coinciding. If $\theta'_{lm\ldots p} = \theta_{lm\ldots p}$, then from (2), in view of (1), it follows that

$$P(f_{ij\ldots k};\theta_{lm\ldots p}) = 1 \tag{3}$$

Due to the well-known features of measure (L), all the usual properties of probability and the conventional calculation methods follow from (2).

In probabilistics a *unidimensional random variable* is defined as a property $t_l \in T$ whose values are real numbers. A property $t_{lm\ldots p} = (t_l, t_m, \ldots, t_p)$, formed of k unidimensional random variables $t_l, t_m, \ldots, t_p$, is called a *k-dimensional random variable*. If values of a random variable are $\varphi(f_l, f_m, \ldots, f_p)$, where $f_l, f_m, \ldots, f_p$ are values of the respective random variables, we denote it by $\varphi(t_l, t_m, \ldots, t_p)$ and call it a *function of random variables*. "Randomness" of a random variable, of course, reveals itself when it undergoes random tests only.

The notion of *state*, widely used in physics, can be introduced immediately into probabilistics. We shall say that an abstract object $f_{ij\ldots k}$ is the abstract object f in a *state* determined by the value $f_{ij\ldots k}$ of the property $t_{ij\ldots k}$ (the state is also denoted by $f_{ij\ldots k}$). The function

$$F(f_{ij\ldots k}; x_l, x_m, \ldots, x_p) = P(f_{ij\ldots k}; f_l \le x_l, f_m \le x_m, \ldots, f_p \le x_p)$$

will be called the *distribution function* of the random variable $t_{lm\ldots p}$ for the state $f_{ij\ldots k}$. If there exists a nonnegative function $\rho(f_{ij\ldots k}; x_l, x_m, \ldots, x_p)$ such that for any domain $\theta'_{lm\ldots p} \subseteq \theta_{lm\ldots p}$ the equality

$$P(f_{ij\ldots k};\theta'_{lm\ldots p}) = \int_{\theta'_{lm\ldots p}} \rho(f_{ij\ldots k}; x_l, x_m, \ldots, x_p)\, d\tau \tag{4}$$

is satisfied ($d\tau = dx_l\, dx_m \cdots dx_p$), we call it the *probability density* of the random variable $t_{lm\ldots p}$ for the state $f_{ij\ldots k}$. From (3) it follows that

$$\int_\Gamma \rho(f_{ij\ldots k}; x_l, x_m, \ldots, x_p) = 1 \tag{5}$$

where $\Gamma = \theta_{lm\ldots p}$.

In the most general case the probability distribution of a random variable $t_{lm\ldots p}$ for the state $f_{ij\ldots k}$ is determined by the pertinent distribution function. We shall say that a mathematical quantity κ *describes* the state $f_{ij\ldots k}$ with respect to the random variable $t_{lm\ldots p}$ if κ uniquely determines $F(f_{ij\ldots k}; x_l, x_m, \ldots, x_p)$. It is evident that if κ_1 describes a certain state and κ_2 uniquely determines κ_1, then κ_2 also describes this state.

One can describe a state $f_{ij\ldots k}$ with respect to a random variable $t_{lm\ldots p}$ by the probability density or by the well-known *characteristic function*, for both uniquely determine the distribution function. There may be other mathematical quantities for describing states, but of special interest and importance are the following two: *state vector* $\boldsymbol{\psi}$ and *density operator* $\hat{W}$ (in what follows the state designation $f_{ij\ldots k}$ is omitted).

The state vector is introduced, proceeding from the fact that the function

$$\psi(x_l, \ldots, x_p) = \rho^{1/2}(x_l, \ldots, x_p)e^{i\eta(x_l, \ldots, x_p)} \tag{6}$$

where η is any real function, uniquely determines $\rho(x_l, \ldots, x_p)$, and hence also describes the state with respect to the random variable $t_{l\ldots p}$. Further, the mathematical expectation of some function $\varphi(t_l)$ of a random variable t_l,

$$M\varphi(t_l) = \int_\Gamma \varphi(x_l)\rho(x_l, \ldots, x_p)\, d\tau \tag{7}$$

can be represented, in view of (6), as

$$M\varphi(t_l) = \int_\Gamma \int_{\Gamma'} \varphi(x_l')\, \delta(\tau' - \tau)\, \psi(x_l', \ldots, x_p')\psi^*(x_l, \ldots, x_p)\, d\tau'\, d\tau \tag{8}$$

and then rewritten as

$$M\varphi(t_l) = (\varphi(\hat{t}_l)\boldsymbol{\psi}, \boldsymbol{\psi}) \tag{9}$$

where $\hat{t}_l$ is the operator corresponding to the random variable t_l, and $\boldsymbol{\psi}$ is a vector. Then (5) yields

$$(\boldsymbol{\psi}, \boldsymbol{\psi}) = 1 \tag{10}$$

which means that $\boldsymbol{\psi}$ is normalized to unity. Since the spectrum of $\hat{t}_l$ coincides with the set θ_l whose elements are real numbers (by definition), $\hat{t}_l$ is a Hermitian operator. The vector $\boldsymbol{\psi}$ uniquely determines the function $\psi(x_l, \ldots, x_p)$, representing it in the basis of eigenvectors of the Hermitian operators $\hat{t}_l, \ldots, \hat{t}_p$. Hence, it also describes the state with respect to the random variable $t_{lm\ldots p}$.

To the vector $\boldsymbol{\psi}$ there can be assigned the operator $\hat{W} = \hat{W}_{(l,\ldots,p)}$, whose kernel in the x representation has the form

$$W(x_l, \ldots, x_p; x_l', \ldots, x_p') = \psi(x_l, \ldots, x_p)\psi^*(x_l', \ldots, x_p') \tag{11}$$

Since the operator $\hat{W}$ uniquely determines $\rho(x_l, \ldots, x_p)$, for in the x representation $W(x_l, \ldots, x_p; x_l, \ldots, x_p) = \rho(x_l, \ldots, x_p)$, it describes, too, the state with respect to the random variable $t_{l\ldots p}$. Now (8) can be rewritten as

$$M\varphi(t_l) = \mathrm{Tr}[\varphi(\hat{t}_l)\hat{W}] \tag{12}$$

with Tr $\hat{W} = 1$.

Introduction of a state vector or a density operator for the description of a state entails the introduction of a Hermitian operator for every random variable. It follows from (9) that an eigenvector of the operator $\hat{t}_l$, which corresponds to its eigenvalue $\alpha \in \theta_l$, describes a state, for which the random variable t_l has a definite value $f_l = \alpha$.

One of the tasks of probabilistics is solving probabilistic problems. The solution of ordinary probabilistic problems requires generally the construction of a set adequate to the set of concrete objects under consideration. This way has been called by me the *classical method.* But this method is inapplicable to many problems in physics. There is, however, another method which I have named the *quantum approach*. When employing it, a state of an abstract object is described with respect to some random variable by a state vector or density operator defined in the corresponding space. It should be emphasized that *both methods belong to probabilistics itself* (Mayants, 1973, 1977, 1984), but their application in physics is of a special interest.

4. PROBABILISTIC PHYSICS

Probabillistic physics deals with the probabilistic treatment of physical systems. Various separate disciplines, such as CSP, QP, statistical hydrodynamics, kinetics of physical and chemical processes, etc., are thus particular domains of probabilistic physics. In probabilistic physics concrete objects and abstract objects are, respectively, concrete physical systems and abstract physical systems. Properties of objects are, in particular, various physical quantities, including coordinates and time. Since values of these quantities are real numbers, physical quantities prove to be random variables by definition. A state of an abstract physical system is determined by definite values of certain physical quantities and can be described in any of the above ways.

One of the main tasks of probabilistic physics is solving pertinent probabilistic problems. Different domains of it may require different ways of doing this. The classical method and the quantum approach are of immediate interest to us now. The starting point of the former is the construction of a set $A^{(a)}$ adequate to the corresponding set A of concrete physical systems. This method was actually used by Gibbs in constructing classical statistical mechanics (ensemble method). The starting point of the latter is finding Hermitian operators for various physical quantities and calculating their eigenvectors and eigenvalues.

It follows from all the statistical experiments we know that the classical method is suitable for "classical" systems consisting of particles of large enough mass, whereas it is unsuitable for "microsystems." We find that the quantum approach alone works for microsystems, but it should also be applicable to classical systems. Since the operators for random variables are the same for both cases, quantum mechanical probability distributions *must* go over into the corresponding classical statistical probability distributions when passing from the former to the latter. Whenever this assertion can be checked, it proves true (Mayants, 1973, 1977, 1984), which means that the "classical" limit of QP is CSP.

5. CSP, QP

The application of the classical method to a closed classical mechanical system immediately leads to Gibbs' microcanonical distribution. When using the quantum approach we arrive at QP. The operators for coordinates and momenta can easily be chosen, $\hat{E} = i\hbar\, \partial/\partial t$ corresponding to energy. The form of the operator $\hat{H}$ for the Hamiltonian function depends on the physical system. In a probabilistic treatment of a mechanical system, the equality $E = H$ for a concrete conservative system should be replaced by the equality of the mathematical expectations of the random variables E and H, which is

$$(\hat{E}\psi, \psi) = L(\hat{H}\psi, \psi) \tag{13}$$

But (13) is satisfied if

$$\hat{H}\psi = \hat{E}\psi \tag{14}$$

is valid, which is the Schrödinger equation.

This article is a brief review intended to give some idea of how QP as a whole science logically emerges from KMP. A more detailed exposition can be found elsewhere (Mayants, 1984). A realistic explanation of wave–corpuscle duality (Mayants, 1989) and of the experimental failure of Bell's inequalities (Mayants, 1991) may also be of interest to many.

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